79-4037

Wing Rock Due to Aerodynamic Hysteresis

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An analysis is presented using control theory concepts to show that aerodynamic hysteresis of the form of relay action can lead to lateral-directional limit cycle motions. These limit cycle motions are usually described in a colloquial sense as airframe wing rock. The aerodynamic details leading to wing-rock oscillations will vary with airframe configuration, and usually the exact description of the aerodynamic nonlinearity is difficult to obtain or predict. The purpose of the studies described herein is to promulgate a flight mechanics analysis technique which by virtue of its simplicity offers the analyst an insight into potential candidate aerodynamic mechanisms.

Nomenclature

A = plant matrix
B = plant control matrix
b = wing span, ft
C_l = rolling moment/ (qSb)
C_n = yawing moment/ (qSb)
C_{l_β} = $\partial C_l / \partial \beta$
C_{l_p} = $\partial C_l / \partial (pb/2U)$
C_{l_r} = $\partial C_l / \partial (rb/2U)$
$C_{l_{\delta a}}$ = $\partial C_l / \partial \delta a$
C_{n_β} = $\partial C_n / \partial \beta$
C_{n_p} = $\partial C_n / \partial (pb/2U)$
C_{n_r} = $\partial C_n / \partial (rb/2U)$
I = unit diagonal matrix
I_x = moment of inertia in roll, slug-ft^2
I_z = moment of inertia in yaw, slug-ft^2
L_β = $(qSb) C_{l_\beta} / I_x$, s^{-2}
L_p = $(qSb^2) C_{l_p} / (2I_x U)$, s^{-1}
L_r = $(qSb^2) C_{l_r} / (2I_x U)$, s^{-1}
N_β = $(qSb) C_{n_\beta} / I_z$, s^{-2}
N_p = $(qSb^2) C_{n_p} / (2I_z U)$, s^{-1}
N_r = $(qSb^2) C_{n_r} / (2I_z U)$, s^{-1}
p = rate of roll, rad/s
q = $\rho U^2/2$ = dynamic pressure, psf
r = rate of yaw, rad/s
S = wing area, ft^2
T = period of oscillation, s
t = time, s
U = aircraft velocity, fps
u = plant control input column vector
x = state variable column vector
β = angle of sideslip, rad
δa = aileron angle, rad
ρ = air density, slug/ft^3
φ = angle of bank, rad
ω = circular frequency, s^{-1}
λ = eigenvalue, s^{-1}
ψ = angle of yaw, rad
Φ = transition matrix
$\Delta(-)$ = incremental change of $(-)$
$\{-\}$ = column vector array
$[-]$ = matrix array
$[-]^{-1}$ = inverse of a square matrix

Superscript
$(\dot{\ })$ = $d(-)/dt$

Received July 5, 1978; revision received Aug. 31, 1978.
Index categories: Handling Qualities, Stability and Control; Aerodynamics.
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Introduction

THE term "wing rock" has been used to describe motion that is experienced by certain aircraft when operating at angles of attack either near to stall onset or when flow separation effects are evident by aircraft buffeting. The motion has been observed in flight, but has been difficult to explain because of its similarity to a lightly damped Dutch-Roll mode. The evidence suggests that the wing-rock motion is a limit-cycle oscillation wherein the amplitude and period of the motion is solely a result of aerodynamic nonlinearities. This is a contrast to the response of a lightly damped Dutch-Roll mode where the amplitude is determined by the initial conditions. The presence of mechnical hysteresis in stability augmentation systems can also give rise to limit-cycle motions, and this situation should not be confused with either the Dutch-Roll or aerodynamic hysteresis effects.

The aerodynamic nonlinearity contributing to the wing-rock motion is difficult to identify from flight test results since most flight test dynamic analysis techniques, including aircraft parameter identification, are oriented toward a linearized model of the aircraft. Presumably a nonlinear model could be assumed for parameter identification, but a question of uniqueness would arise since it might be possible for several completely different nonlinear models of the plant to provide equally valid time history matches with the actual flight data. A logical step at this time would be the recognition of wing rock as a limit-cycle oscillation followed by a consideration of some possible aerodynamic causes.

Ross[1] has shown that a wing-rock limit cycle can exist when the nonlinearity is a cubic behavior in either yawing or rolling moment with respect to sideslip angle. Ross used the method of slowly varying parameters (Krylov-Bogoliubof method) in order to demonstrate the limit cycle. In the method of slowly varying parameters, the cross-coupled nonlinear differential equations are reorganized into a single, higher-order differential equation, and the influence of nonlinear aerodynamic coefficients are analyzed by stability criteria in a manner somewhat similar to the "Galloping Transmission Line" analysis by Parkinson and Smith.[2]

The analysis described herein is an application of control theory as described by Ogata[3] where the cross-coupled second-order differential equations are cast into state variable form to yield a larger set of first order differential equations. This technique is convenient for linear system analysis and quite adaptable to hysteresis situations from relay-type inputs. The advantage gained from classical procedures in treating nonlinear systems is that physical concepts or principles become understandable and a broader insight is acquired. Direct numerical (or analog) system simulations are useful for exploring the traits of a particular vehicle when more specific aerodynamic definitions are available and results are required for correlation with companion flight data.

Wind-tunnel tests upon aircraft models frequently show that the rolling moments measured on a symmetric model when yaw angle is zero and angle of attack is increased are approximately zero valued until attack angles near to stall are reached. Rolling moments tend to be erratic at or near stall, and it is customary to attribute these traits to slight asymmetries in either the model or tunnel flow with a resulting early stall of one wing compared to the other. A recent wind-tunnel test[4] upon a sting-mounted model having a strain gage balance with appropriate electronic filtering used the test technique of conducting a slow, continuous yaw angle sweep through a complete cycle of yaw motion. When the cyclic yaw sweep was conducted at or near to stall, both rolling and yawing moments showed incremental shifts in value that appeared to be related to the direction of the slow yaw sweep. The curves of rolling moment, when plotted vs yaw angle, gave evidence of aerodynamic hysteresis.

Aerodynamic hysteresis in the form of lift and pitching moment variations in the neighborhood of stall has been of much interest in rotary wing dynamics as typically pointed out by Crimi.[5] A mechanism for explaining aircraft "stall porpoising" has been proposed by Schoenstadt[6] using a nonlinear relay model on the lift curve stall break. The solution by Schoenstadt achieved its elegance by simplifying the longitudinal equations of motion in the neighborhood of stall to a second-order plant. One degree more of complication is introduced when considering aircraft lateral-directional response, since a roll subsidence mode needs to be included with the Dutch-Roll mode in order to explain wing rock by nonlinear relay action.

Theory

The analysis to be described here will include some simplifications since the intent is to use a state variable formulation in an illustrative sense for demonstrating a wing-rock limit-cycle motion due to aerodynamic hysteresis. We shall assume that the lateral-directional oscillation of the aircraft occurs while the aircraft is on a straight flight path, hence yaw and sideslip angles are related by

$$\psi = -\beta \tag{1}$$

Removal of the constraint implied by Eq. (1) would add a side force equilibrium equation to the following formulation. The next assumption will be that the aircraft is operating at a fixed angle of attack with all control surfaces fixed. Therefore,

$$\dot{r}I_z = \text{yawing moment} = (qsB)\,[C_{n_\beta}\beta + (b/2U)(C_{n_r}r + C_{n_p}p)] \tag{2}$$

and

$$\dot{p}I_x = \text{rolling moment} = (qSb)\,[C_{l_\beta}\beta + (b/2U)(C_{l_r}r + C_{l_p}p)] \tag{3}$$

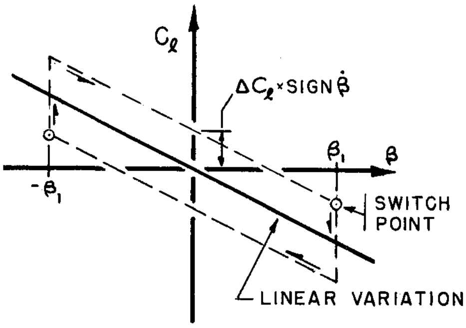

Fig. 1 Aerodynamic roll moment hysteresis.

Since $r = d\psi/dt$, etc., we may simplify Eqs. (2) and (3) in light of Eq. (1) to

$$\ddot{\beta} = -N_\beta\beta + N_r\dot{\beta} - N_p p$$

and

$$\dot{p} = L_\beta\beta - L_r\dot{\beta} + L_p p$$

or in state variable form, the freely vibrating system may be dynamically described as

$$\{\dot{x}\} = [A]\{x\}$$

which corresponds to a matrix array representation of

$$\begin{Bmatrix} \dot\beta \\ \ddot\beta \\ \dot p \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -N_\beta & N_r & -N_p \\ L_\beta & -L_r & L_p \end{bmatrix} \begin{Bmatrix} \beta \\ \dot\beta \\ p \end{Bmatrix} \tag{4}$$

$\{x\}$ is the state vector whose components are sideslip angle (β), sideslip angular velocity ($\dot\beta$), and roll rate (p). The plant matrix $[A]$, of order 3×3, is composed of the dimensional stability derivative parameters as defined by McRuer et al. As will be pointed out, Eq. (4) is quite adequate to illustrate the principles contributing to wing rock.

The solution to Eq. (4) along with the prescribed initial conditions forms the linear system Dutch Roll and roll rate responses. In state variable form, the transient response solution may be expressed as[3]

$$\{x(t)\} = [\Phi(t)]\{x(0)\} \tag{5}$$

where $[\Phi(t)]$ = state transition matrix. The equations of motion for the dynamic system response when control is applied is given by

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} \tag{6}$$

where the plant control matrix $[B]$ would reflect inputs to yaw and roll moment, typically from control by aileron or rudder deflections, which would be contained as components in the $\{u\}$ vector. The complete solution to the linear, time-invariant Eq. (6) becomes

$$\{x(t)\} = [\Phi(t)]\{x(0)\} + [\Phi(t)]\int_0^t [\Phi(-\tau)]\,[B]\{u(\tau)\}\,d\tau \tag{7}$$

In developing the procedure for solving the airframe response due to a hysteresis type of aerodynamic input by the relationships implied in Eq. (7), let us next visualize an initial form of aerodynamic hysteresis. Figure 1 illustrates the assumed form of aerodynamic hysteresis wherein the hysteresis action provides an incremental modification to the airframe dihedral effect term dependent upon the sign of $\dot\beta$. In other words, the incremental rolling moment ΔC_l retains a constant magnitude but switches sign depending upon the sign of the $\dot\beta$ term. The relay action can also occur in yawing moment and conceivably could be dependent upon the sign of other state variables such as β or p.

The motion of the system will in general correspond to a periodic variation of the state variables. The amplitude variation of sideslip angle, as shown in Fig. 1, may be assumed as being bounded such that

$$-\beta_l \le \beta \le \beta_l$$

The hysteresis term is assumed as an additional roll moment term which depends for its value upon the sign of sideslip angle rate. Consequently, the rolling moment due to sideslip, as shown on Fig. 1, may be expressed as

$$C_l(\beta) = C_{l_\beta}\beta + \Delta C_l \operatorname{sgn}\dot{\beta} \tag{8}$$

Using the property that the system is time invariant, it is convenient to establish the time origin when $\beta = \beta_l$. The variation of β (or x_l) during the advent of a limit cycle is portrayed in Fig. 2. In parallel, the switching action on the roll moment increment is sketched for concept clarity.

The periodic nature of the assumed limit cycle requires that

$$\{x(t)\} = \{x(t+T)\}$$

where T = period of the limit cycle. The hysteresis switching dependence upon $\dot{\beta}$ (or x_2) in conjunction with the convenient choice of time origin implies that

$$x_2(0) = x_2(T/2) = x_2(T) = \dot{\beta} = 0$$

A symmetry argument upon the state variable can be made for the situation of a limit cycle with $\dot{\beta}$ switching to require that

$$\{x(T/2)\} = -\{x(0)\} \tag{9a}$$

and

$$x_2(T/2) = x_2(0) = 0 \tag{9b}$$

Assume now that the limit cycle oscillation has become stabilized (i.e., all startup transients have decayed), then the relay action for the response referenced to the time origin of $t=0$ can be described by

$$-2\Delta C_l 1(t) + 2\Delta C_l 1(t-T/2) - \ldots \tag{10}$$

where $1(t)$ is the unit step function.

During the first half-cycle of the oscillation, the forcing function portion of Eq. (6) can be expressed as

$$[B]\{u\} = (\Delta L)\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}; \text{ for } 0<t<T/2 \tag{11}$$

where

$$(\Delta L) = -2(qSb)\Delta C_l / I_x$$

Note that adverse or proverse yaw coupling can be introduced into the u column vector by incorporating a nonzero u_2 term. Using the hysteresis form of the forcing function as stated in Eq. (11), the solution of the response at $t = T/2$ may be evaluated by applying Eq. (7):

$$\left\{x\left(\frac{T}{2}\right)\right\} = \left[\Phi\left(\frac{T}{2}\right)\right]\{x(0)\} + (\Delta L)\left[\Phi\left(\frac{T}{2}\right)\right]\int_0^{T/2}[\Phi(-\tau)]\,\mathrm{d}\tau \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \tag{12}$$

The integral expression in Eq. (12) can be deduced quite readily by noting some properties of the transition matrix, namely,

$$\frac{\mathrm{d}}{\mathrm{d}t}[\Phi(t)] = [A][\Phi(t)] = [\Phi(t)][A]$$

$$[\Phi(t_1 + t_2)] = [\Phi(t_1)][\Phi(t_2)]$$

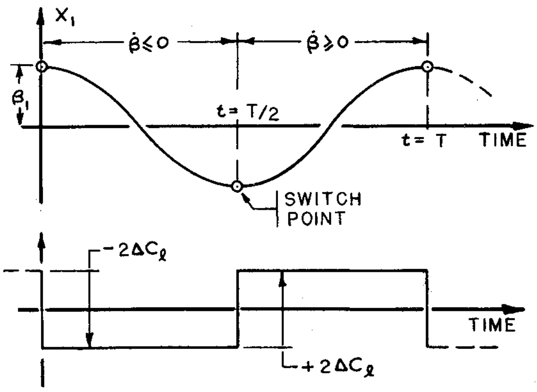

Fig. 2 Switch action during limit cycle.

and

$$[\Phi(0)] = [I] = \text{diagonal identity matrix}$$

By combining the transition matrix properties with the symmetry constraint conditions from Eq. (9), one obtains

$$\left\{x\left(\frac{T}{2}\right)\right\} = -(\Delta L)[A]^{-1}\left[I + \Phi\left(\frac{T}{2}\right)\right]^{-1}\left[I - \Phi\left(\frac{T}{2}\right)\right]\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \tag{13}$$

The numerical solution of the preceding equation for the semiperiod of the limit-cycle oscillation may be obtained iteratively while observing the $x_2(T/2)$ component for satisfaction of the zero value requirement. A sample solution is presented in the next section. Although the solution of Eq. (13) does not appear to depend upon the sign or magnitude of the roll moment hysteresis, it will be shown in the next section that the concept of airstream energy removal during a limit cycle fixes the sign.

Analysis

Although it would be more complete if the analysis were made for an aircraft with known wing-rock traits such as the A-4, F-4, or F-5 vehicles followed by a comparison with flight test measurements, access to such information is somewhat restrictive. Therefore, consistent with the purpose of this analysis, we shall consider a situation using representative aircraft stability derivative and inertial properties. The aircraft considered was the F-94A in the landing configuration as defined by Blakelock.[8] For this assumed situation, we have

U	$= 198$ fps	S	$= 239\ \mathrm{ft}^2$
q	$= 46.6$ psf	b	$= 37.3$ ft
I_x	$= 7.169 \times 10^3\ \mathrm{slug\text{-}ft}^2$	I_z	$= 33.01 \times 10^3\ \mathrm{slug\text{-}ft}^2$
C_{l_β}	$= -0.0487\ \mathrm{rad}^{-1}$	C_{n_β}	$= 0.105\ \mathrm{rad}^{-1}$
C_{l_p}	$= -0.450\ \mathrm{rad}^{-1}$	C_{n_p}	$= -0.053\ \mathrm{rad}^{-1}$
C_{l_r}	$= 0.278\ \mathrm{rad}^{-1}$	C_{n_r}	$= -0.210\ \mathrm{rad}^{-1}$
$C_{l_{\delta_a}}$	$= -0.0916\ \mathrm{rad}^{-1}$		

For these assumed values, the plant matrix of Eq. (4) becomes

$$[A] = \begin{bmatrix} 0 & 1 & 0 \\ -1.3214 & -0.2491 & 0.0629 \\ -2.8220 & -1.5170 & -2.4557 \end{bmatrix} \tag{14}$$

The state variable equations were solved using a computer code[9] available on the Naval Postgraduate School's IBM 360 digital computer. The eigenvalues of Eq. (4) using the plant as defined by Eq. (14) were

$$\lambda_1 = a = -2.4473 \text{ sec}^{-1} \tag{15a}$$

$$\lambda_{2,3} = e \pm ic = -0.1287 \pm i\ 1.1755 \text{ sec}^{-1} \tag{15b}$$

The Dutch-Roll mode corresponds to a system with a damping level and undamped natural frequency of approximately 10.9% of critical and 1.1826 rad/s, respectively.

The state transition matrix, $[\Phi(t)]$, which is a linear combination of the eigenvectors, was

$$[\Phi(t)] = [R]\exp(at) + \{[F]\cos(ct) + [G]\sin(ct)\}\exp(et) \tag{16}$$

where

$$[R] = \begin{bmatrix} 0.0114 & 0.0012 & 0.0093 \\ -0.0279 & -0.0030 & -0.0228 \\ 1.2146 & 0.1318 & 0.9916 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 0.9886 & -0.0012 & -0.0093 \\ 0.0279 & 1.0030 & 0.0228 \\ -1.2146 & -0.1318 & 0.0084 \end{bmatrix}$$

$$[G] = \begin{bmatrix} 0.1320 & 0.8531 & 0.0184 \\ -1.1791 & -0.1084 & 0.0086 \\ -0.0050 & -1.0305 & -0.0236 \end{bmatrix}$$

The complete set of solutions, as prescribed by Eq. (13), was programmed on an HP 9830 calculator. The results for a unit value of roll hysteresis parameter ΔL were period: $T = 1.0473\ T_d$ where

$$T_d = \text{Dutch-Roll period} = 5.345 \text{ s} \tag{17}$$

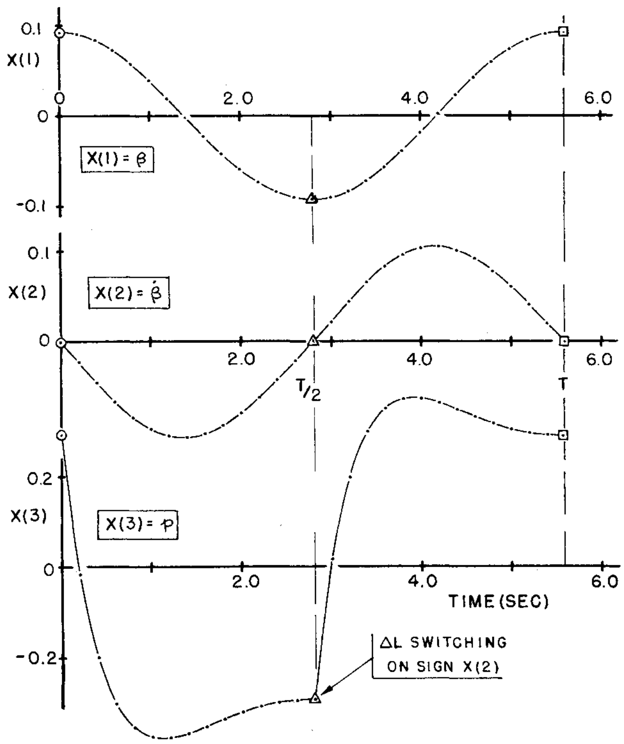

Fig. 3 Limit cycle time history.

state vector at $t=0$ for $\Delta L = -1$:

$$\{x(0)\} = \begin{Bmatrix} \beta(0) \\ \dot\beta(0) \\ p(0) \end{Bmatrix} = \begin{Bmatrix} +0.0921 \text{ rad} \\ 0 \\ +0.2949 \text{ rad/s} \end{Bmatrix} \tag{18}$$

In addition to obtaining an estimate of the limit cycle period and the state vector at a switch time, $t=0$, the conciseness of the control theory solution made possible time history calculations on the HP 9830 calculator of the airframe response during the limit cycle and also the linear system response during the Dutch-Roll motion. Figure 3 is a one-cycle time history of the limit cycle. Worthy of note is that the sideslip related terms, x_1 and x_2, show a near harmonic response. In contrast, the roll rate x_3 time history shows the effect of roll moment switching and has the approximate appearance of a square wave after being processed by a low-pass filter. The low-pass filter action is a byproduct of the airframe's roll subsidence mode responding to a step input change of roll moment.

The time history of x_3 on Fig. 3 allows one to deduce that the sign on the roll moment increment must result in the

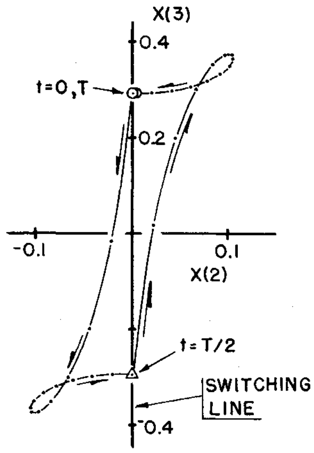

Fig. 4 Limit cycle trajectory.

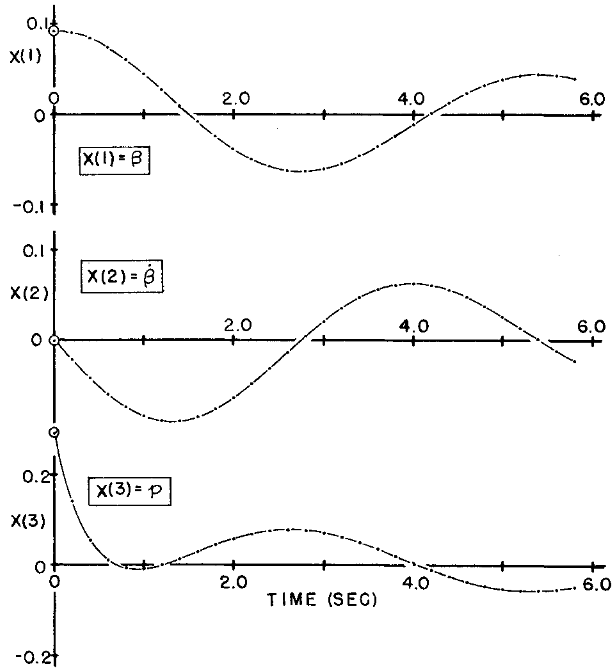

Fig. 5 Dutch-Roll time history.

extraction of energy from the freestream, i.e., work per cycle:

$$W = \tfrac{1}{2} I_x \oint (\Delta L) x_3 \, dt > 0$$

Therefore, for the first half-period of the limit cycle shown on Fig. 3, the ΔL increment should be negative in sign followed by a switching to a positive increment.

A companion state variable trajectory of x_3 vs x_2 for the limit cycle is shown on Fig. 4. The action of the trajectory crossing the $x_2 = 0$ switching line becomes quite evident.

The linear system time history response to a set of initial conditions at $t = 0$ is shown on Fig. 5 for comparison purposes. Since the system is lightly damped, the time histories of x_1 and x_2 are quite similar to those shown on the limit-cycle time history, Fig. 3. This similarity in behavior for a few cycle encounter of a limit cycle during flight could provide one with the impression of having experienced a Dutch Roll instead of a limit cycle. The roll rate time histories are significantly different, but pilot perception is tuned more to bank angle changes rather than roll rate changes, hence the distinction between the two types of roll rate time histories might be hard to note during a flight encounter.

The companion state variable trajectory of x_3 vs x_2 for the linear response is shown on Fig. 6. Initiation of the trajectory from an initial condition results in the classical type of state vector trajectory for a damped linear system to the static equilibrium point, which may be described as asymptotic stability in the sense of Lyapunov.[3]

As an added aid to visualizing the magnitude of the wing rock described by these analyses, let us assume that the roll hysteresis corresponds to about $\pm 10\%$ of available aileron control power, and that typically 20 deg of deflection corresponds to full aileron control. Using the tabulated value of $C_{l_{\delta}}$, we can estimate a value of $\Delta L = -0.37$. Continuing the process, we may estimate that the amplitude of the sideslip angle, roll rate, and bank angle would be appproximately: $|\beta(t)| = 1.96$ deg, $|p(t)| = 6.26$ deg/s, and $|\varphi(t)| = 17.5$ deg. Since roll rate is approximately in phase with sideslip angle rate (Fig. 3), we would expect that bank angle and sideslip angle would also be approximately in phase.

The state transition matrix, which is a linear combination of the eigenvectors, allows one to identify the Dutch-Roll mode. For this example, we find that the eigenvectors, when normalized with respect to sideslip angle, are given by

$$\begin{aligned} \beta_d &= 1.00\,; && \text{phase angle} = 0 \text{ deg} \\ \beta_d &= 1.183; && \text{phase angle} = +96.3 \text{ deg} \\ p_d &= 1.218; && \text{phase angle} = +187.4 \text{ deg} \\ \varphi_d &\cong 1.030; && \text{phase angle} \cong +91.1 \text{ deg} \end{aligned}$$

Observations from this analysis include the following;

1) Limit cycle period does not differ significantly from the Dutch-Roll period, and does not change with magnitude of roll moment hysteresis.

2) The limit cycle amplitudes are directly related to the magnitude of the roll moment relay action.

3) The sign of ΔL during switching can be determined from the requirement that a limit cycle with a dissipative system requires extraction of energy from the freestream.

4) During a limit cycle, the first harmonic of the roll rate response is approximately in phase with the sideslip angle rate while it is approximately ninety degrees out of phase during a Dutch-Roll oscillation.

5) In addition to the phase differences between the limit cycle and Dutch-Roll oscillations, the relative amplitude ratios of bank angle to sideslip angle show distinctive differences for this example, with the ratio during a limit cycle being about 8½ times greater than that occurring during a Dutch-Roll oscillation. The emphasis on bank angle variation would lead one to describe the oscillation as a wing rock.

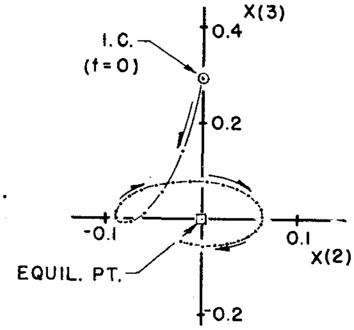

Fig. 6 Dutch-Roll trajectory.

Concluding Remarks

The existence of a lateral-directional aircraft limit-cycle oscillation due to an aerodynamic hysteresis corresponding to a roll moment relay action that is dependent upon the sign of sideslip velocity has been demonstrated. The use of state variable notation allowed the limit-cycle motion to be expressed in a concise and clear manner.

Although bank angle was not a state variable in the analysis and hence could only be estimated from roll rate, the inclusion of this term by the introduction of a side force equilibrium equation is practicable for a more detailed analysis. However, since the purpose of the analysis described here was to illustrate a concept, the only equations used were those absolutely necessary for achieving that goal.

Finally, it is hoped that these analyses will make clearer that interpreting an actual aircraft time history as representing a Dutch-Roll mode with exactly zero damping may be misleading since a motion with very similar appearance could be a limit cycle due to nonlinear yawing and rolling moments as posed by Ross[1] or due to aerodynamic relay or hysteresis actions as suggested here. The direct problem of determining the time response of an aircraft due to forcing functions, be they linear or nonlinear, is tractable. The inverse problem of identifying the plant from the aircraft time histories (known as aircraft parameter identification) is much more difficult because of the uncertainties in estimating types of plant nonlinearities.

References

[1] Ross, A.J., "Investigation of Nonlinear Motion Experienced on a Slender-Wing Research Aircraft," *Journal of Aircraft*, Vol. 9, Sept. 1972, pp. 625-631.

[2] Parkinson, G.V. and Smith, J.D., "The Square Prism as an Aeroelastic Non-Linear Oscillator," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 17, May 1964, pp. 225-239.

[3] Ogata, K., *Modern Control Engineering*, Prentice-Hall, Englewood Cliffs, N.J., 1970.

[4] Ottensoser, J., "High Angle of Attack Aerodynamic Data for a 0.10 Scale A-7 Model Evaluated in the 7 by 10 Foot Transonic Wind Tunnel for Investigating the Stall Departure Phenomena, Phase I," Naval Ship Research and Development Center TM-16-76-16, Oct. 1975.

[5] Crimi, P., "Analysis of Helicopter Rotor Blade Stall Flutter," *Journal of Aircraft*, Vol. 11, July 1974, pp. 407-413.

[6] Schoenstadt, A.L., "Nonlinear Relay Model for Post-Stall Oscillations," *Journal of Aircraft*, Vol. 12, July 1975, pp. 572-577.

[7] McRuer, D., Ashkenas, I., and Graham, D., *Aircraft Dynamics and Automatic Control*, Princeton Univ. Press, Princeton, N.J., 1973.

[8] Blakelock, J.K., *Automatic Control of Aircraft and Missiles*, J. Wiley and Sons, New York, 1965.

[9] Melsa, J.L. and Jones, S.K., *Computer Programs for Computational Assistance in the Study of Linear Control Theory*, McGraw-Hill, New York 1973.